

Roll No.

92007

B. Sc. 3rd Semester (Mathematics)

Examination – November, 2014

PARTIAL DIFFERENTIAL EQUATIONS

Paper : BM-232

Time : Three Hours] [Maximum Marks : 40

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

प्रश्नों के उत्तर देने से पहले परीक्षार्थी यह सुनिश्चित कर लें कि उनको पूर्ण एवं सही प्रश्न-पत्र मिला है। परीक्षा के उपरान्त इस संबंध में कोई भी शिकायत नहीं सुनी जायेगी।

Note : Attempt any *five* questions, selecting *one* question

from each Unit. Unit - V is *compulsory*

प्रत्येक इकाई से कम से कम एक प्रश्न चुनते हुए, कोई पाँच प्रश्न कीजिये। इकाई - V अनिवार्य है।

92007-19,500-(P-7)(Q-9)(14)

P. T. O.

UNIT - I**इकाई - I**

1. (a) Find the differential equation of all spheres of fixed radius having centre in xy plane.

xy -तल में केन्द्र वाले स्थिर त्रिज्या के सभी गोलकों के अवतल समीकरण ज्ञात कीजिये।

- (b) Form the differential equations by eliminating arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.

$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$ से स्वेच्छाचारी फलन के विलोपन द्वारा अवकल समीकरण बनाइए।

2. (a) Solve $x(y^2 - z^2)p - y(z^2 + x^2)q = z(x^2 + y^2)$.

हल कीजिये :

$$x(y^2 - z^2)p - y(z^2 + x^2)q = z(x^2 + y^2)$$

- (b) Solve $p = (qy + z)^2$ by using charpit method.

चारपिट की विधि से $p = (qy + z)^2$ को हल कीजिये।

UNIT - II**इकाई - II**

3. (a) Solve $(D^3 - 4D^2 D' + 4D'^2 D) Z = \cos(2x + y)$.

हल कीजिये :

$$(D^3 - 4D^2 D' + 4D'^2 D) Z = \cos(2x + y)$$

(b) Solve $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = (x^2 + y^2)^{4/2}$.

हल कीजिये :

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = (x^2 + y^2)^{4/2}$$

4. (a) Solve $\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial x \partial y} + 3 \frac{\partial^2 z}{\partial y^2} = \sqrt{x+3y}$.

हल कीजिये :

$$\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial x \partial y} + 3 \frac{\partial^2 z}{\partial y^2} = \sqrt{x+3y}$$

(b) Solve $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} - x \frac{\partial z}{\partial x} = \frac{x^3}{y^2}$.

हल कीजिये :

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} - x \frac{\partial z}{\partial x} = \frac{x^3}{y^2}$$

UNIT - III

इकाई - III

5. (a) Solve $y^2 \left(\frac{\partial^2 z}{\partial x^2} \right) - 2xy \left(\frac{\partial^2 z}{\partial x \partial y} \right) + x^2 \left(\frac{\partial^2 z}{\partial y^2} \right)$
 $= \left(\frac{y^2}{x} \right) \frac{\partial z}{\partial x} + \left(\frac{x^2}{y} \right) \frac{\partial z}{\partial y}$

by reducing it to canonical form.

92007-19,500-(P-7)(Q-9)(14) (3)

P. T. O.

$$y^2 \left(\frac{\partial^2 z}{\partial x^2} \right) - 2xy \left(\frac{\partial^2 z}{\partial x \partial y} \right) + x^2 \left(\frac{\partial^2 z}{\partial y^2} \right) = \left(\frac{y^2}{x} \right) \frac{\partial z}{\partial x} + \left(\frac{x^2}{y} \right) \frac{\partial z}{\partial y}$$

को इसके नियमानुसार रूप में समाधान द्वारा हल कीजिये।

- (b) Solve $r - t \cos^2 x + p \tan x = 0$.

हल कीजिये $r - t \cos^2 x + p \tan x = 0$.

6. (a) Classify and reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + 4 \frac{\partial^2 z}{\partial x \partial y} - 5 \frac{\partial^2 z}{\partial y^2} + 6 \frac{\partial z}{\partial x} + 3 \frac{\partial z}{\partial y} - 9z = 0$$

to canonical form.

$$\frac{\partial^2 z}{\partial x^2} + 4 \frac{\partial^2 z}{\partial x \partial y} - 5 \frac{\partial^2 z}{\partial y^2} + 6 \frac{\partial z}{\partial x} + 3 \frac{\partial z}{\partial y} - 9z = 0$$

को इसके नियमानुसार रूप में समाधान कीजिये।

- (b) Solve $r^2 = at$.

हल कीजिये $r^2 = at$.

UNIT - IV

इकाई - IV

7. (a) Find the characteristics of

$$\begin{aligned} & xy \frac{\partial^2 u}{\partial x^2} - (x^2 - y^2) \frac{\partial^2 u}{\partial x \partial y} - xy \frac{\partial^2 u}{\partial y^2} \\ & + y \frac{\partial u}{\partial x} - x \frac{\partial u}{\partial y} - z(x^2 - y^2) = 0 \end{aligned}$$

$$xy \frac{\partial^2 u}{\partial x^2} - (x^2 - y^2) \frac{\partial^2 u}{\partial x \partial y} - xy \frac{\partial^2 u}{\partial y^2} + y \frac{\partial u}{\partial x} - x \frac{\partial u}{\partial y} - z(x^2 - y^2) = 0$$

की विशेषताएँ ज्ञात कीजिये।

- (b) Solve the Cauchy problem described by the equation $\frac{\partial^2 z}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 z}{\partial t^2} = 0, c > 0$ subject to the condition $z(x, 0) = f(x)$ and $\left[\frac{\partial z}{\partial t} \right]_{t=0} = g(x)$.

आरंभिक शर्तों $z(x, 0) = f(x)$ तथा $\left[\frac{\partial z}{\partial t} \right]_{t=0} = g(x)$ के

अधीन, समीकरण

$\frac{\partial^2 z}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 z}{\partial t^2} = 0, c > 0$ द्वारा वर्णित, काउची समस्या को हल कीजिये।

8. (a) Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

subject to the condition $u(x, 0) = u(x, b) = 0$ for $0 \leq x \leq a$ $u(0, y) = 0$ and $u(a, y) = ky(b - y)$ for $0 \leq y \leq b$.

लाप्लास के समीकरण को हल कीजिये :

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ बशर्ते कि

$u(x, 0) = u(x, b) = 0, 0 \leq x \leq a$ के लिये $u(0, y) = 0$

तथा $u(a, y) = ky(b - y), 0 \leq y \leq b$ के लिए

(b) Find the solution of the wave equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \text{ using its method of separation of variables.}$$

चरों के विलगीकरण की विधि का उपयोग करते हुए
तरंग समीकरण

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \text{ का हल ज्ञात कीजिये।}$$

UNIT - V

इकाई - V

9. (a) Eliminate the arbitrary function to form a differential equation for the equation

$$z = e^{ax+by} f(ax+by)$$

$z = e^{ax+by} f(ax+by)$, से स्वेच्छाचारी फलन के विलोपन
द्वारा अवकल समीकरण ज्ञात कीजिये।

- (b) Solve $p + 3q = z + \cot(y - 3x)$.

$p + 3q = z + \cot(y - 3x)$ को हल कीजिये।

- (c) Show that the equations $xp - yq = x$, $x^2 p + q = xz$
are compatible.

दिखाइए कि समीकरण $xp - yq = x$ तथा $x^2 p + q = xz$
संगत है।

(d) Classify the differential equation

$$\frac{\partial^2 z}{\partial x^2} + 6 \frac{\partial^2 z}{\partial x \partial y} + 9 \frac{\partial^2 z}{\partial y^2} = 0$$

अवकल समीकरण का वर्गीकरण कीजिये।

(e) What is the particular integral of $F(D, D') z = f(x, y)$?

$F(D, D') z = f(x, y)$ का विशिष्ट समाकल क्या है ?

(f) Write two dimensional Heal equation.

द्वि आयामी उष्मा समीकरण लिखिए।